COSC 5P05 - Introduction to Lambda-Calculus

Term Test 3

Question 1 (10 marks): Show that $[\![\lambda x : A.(M\ x)]\!]_\Delta = [\![M]\!]_\Delta$ if x is not free in M.

Solution:

$$
\begin{aligned}
[\![\lambda x : A.(M\ x)]\!]_\Delta &= \Lambda([\![(M\ x)]\!]_{\Delta\cup\{x\}} && \text{Sem. Abstr.}\\
&= \Lambda(\text{eval} \circ \langle [\![M]\!]_{\Delta\cup\{x\}}, [\![x]\!]_{\Delta\cup\{x\}}\rangle) && \text{Sem. Appl.}\\
&= \Lambda(\text{eval} \circ \langle [\![M]\!]_\Delta \circ p_1, [\![x]\!]_{\Delta\cup\{x\}}\rangle) && \text{Prop. 7.}\\
&= \Lambda(\text{eval} \circ \langle [\![M]\!]_\Delta \circ p_1, p_2\rangle) && \text{Prop. 6.}\\
&= \Lambda(\text{eval} \circ ([\![M]\!]_\Delta \times \text{id}) \circ \langle p_1, p_2\rangle) && \text{Prop. 3.}\\
&= \Lambda(\text{eval} \circ ([\![M]\!]_\Delta \times \text{id})) && \text{Prop. 1.}\\
&= \Lambda(\text{eval}) \circ [\![M]\!]_\Delta && \text{Prop. 5.}\\
&= [\![M]\!]_\Delta. && \text{Prop. 4.}
\end{aligned}
$$

Question 2 (10 marks): Show that $[\![\langle \text{fst}(M), \text{snd}(M)\rangle]\!]_\Delta = [\![M]\!]_\Delta$ for every λ-term M.

Solution:

$$
\begin{aligned}
[\![\langle \mathrm{fst}(M), \mathrm{snd}(M)\rangle]\!]_\Delta &= \langle [\![\mathrm{fst}(M)]\!]_\Delta, [\![\mathrm{snd}(M)]\!]_\Delta\rangle && \text{Sem. Pair}\\
&= \langle p_1 \circ [\![M]\!]_\Delta, p_2 \circ [\![M]\!]_\Delta\rangle && \text{Sem. Proj.}\\
&= \langle p_1, p_2\rangle \circ [\![M]\!]_\Delta && \text{Prop. 2.}\\
&= [\![M]\!]_\Delta. && \text{Prop. 1.}
\end{aligned}
$$