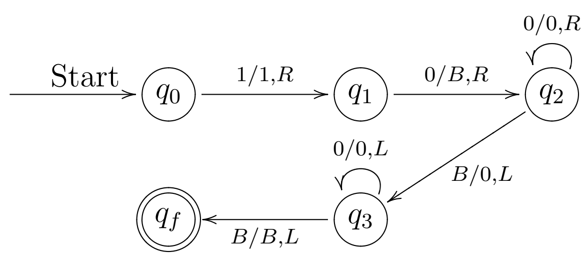
COSC/MATH 4P61 - Theory of Computation

Term Test 3

Question 1: (10 marks) Construct a Turing machine that transforms an initial tape of the form 0^m10^n (m and n 0's with $m, n > 0$ separated by a 1) into $0^m1\_0^n$ (m and n 0's with $m, n > 0$ separated by a 1 followed by a blank). The tape head should be at the 1 before and after the computation. Run your machine on the input 00100.

Solution: We define $M = (\{q_0, q_1, q_2, q_3, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\})$ by:



Tape								State
␣	0	0	1	0	0	␣	␣	q_0
			↑					
␣	0	0	1	0	0	␣	␣	q_1
				↑				
␣	0	0	1	␣	0	␣	␣	q_2
					↑			
␣	0	0	1	␣	0	␣	␣	q_2
						↑		
␣	0	0	1	␣	0	0	␣	q_3
					↑			
␣	0	0	1	␣	0	0	␣	q_3
				↑				
␣	0	0	1	␣	0	0	␣	q_f
			↑					

Question 2: (10 marks) Consider the language

$$L = \{w010w \mid w \in L_d\}.$$

Is this language recursively enumerable? Justify your answer.

Solution: Suppose L would be recursively enumerable, i.e., there is a Turing machine M that accepts L. Then we can construct a Turing machine M' that accepts L_d as follows. Given an input w, M' modifies the input to $w010w$ and then simulates M. If M accepts, then w is in L_d, and our machine M' accepts as well. If M does not accept $w010w$ or runs forever, then M' does the same. The existence of M' is a contradiction to the fact that L_d is not recursively enumerable.