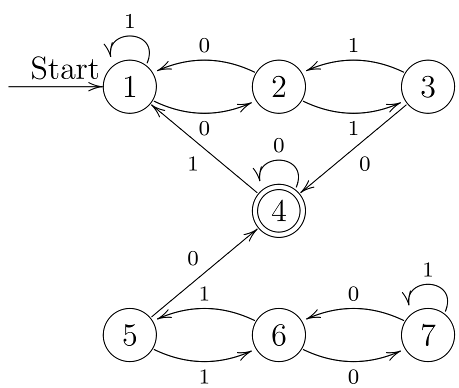
COSC/MATH 4P61 - Theory of Computation

Example Questions Test 2

Question 1: Consider the DFA below. Construct the minimum-state equivalent DFA.



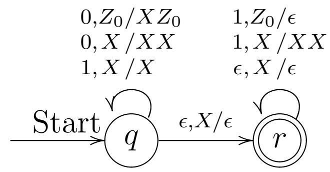
Question 2: Use the Pumping Lemma to show that the language

$$L = \{0^i 1^{2i} \mid i \geq 1\}$$

is not regular.

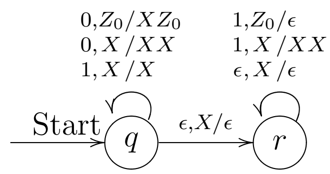
Question 3: Suppose the following PDA $P = (\{q, r\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{r\})$

is given:



Convert P to a PDA P' with $N(P') = L(P)$.

Question 4: Suppose the following PDA $P = (\{q, r\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \emptyset)$ is given:



Convert P to a PDA P' with $L(P') = N(P)$.

Question 5: Convert the grammar

$$S \to 0S1 \mid A$$
$$A \to 1A0 \mid S \mid \epsilon$$

to a PDA that accepts the same language by empty stack.

Question 6: Consider the following grammar:

$$S \to ASB \mid \epsilon$$
$$A \to aAS \mid a$$
$$B \to SbS \mid A \mid bb$$

a. Eliminate all ϵ-productions.

b. Eliminate all unit productions from the resulting grammar in a).

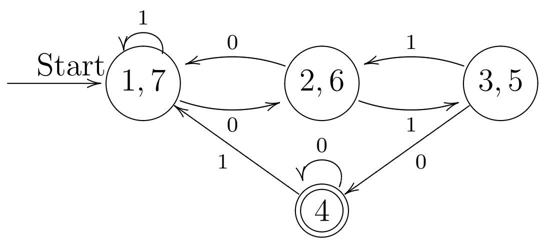
c. Eliminate all useless symbols from the resulting grammar in b).

d. Put the resulting grammar in c) in Chomsky Normal Form.

Solutions

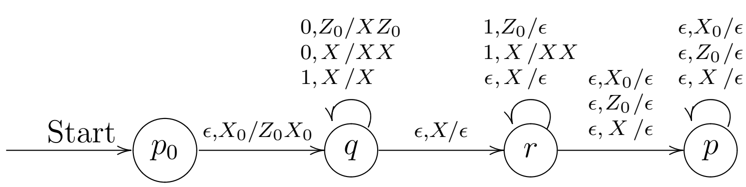
Question 1:

	1	2	3	4	5	6
2	$\times$					
3	$\times$	$\times$				
4	$\times$	$\times$	$\times$			
5	$\times$	$\times$		$\times$		
6	$\times$		$\times$	$\times$	$\times$	
7		$\times$	$\times$	$\times$	$\times$	$\times$

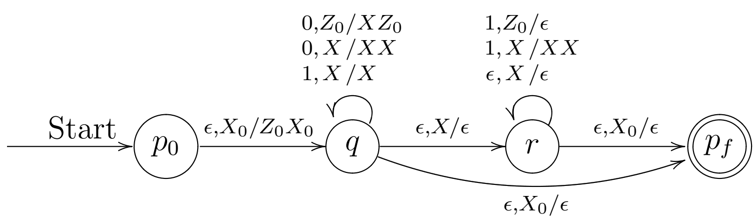


Question 2: Assume L is regular and let n be the constant from the Pumping Lemma. Pick the word $w = 0^n 1^{2n}$. Using the Pumping Lemma we can write w as $w = xyz$ with $|xy| \leq n$. Therefore $y \neq \epsilon$ consists of only 0's. The word $xy^0z = xz$ must be in L, which is a contradiction since it contains $2n$ copies of 1 but less than n copies of 0.

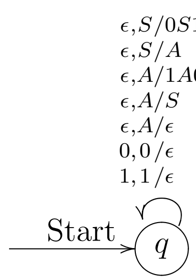
Question 3: The PDA $P' = (\{p_0, p, q, r\}, \{0, 1\}, \{X_0, Z_0, X\}, \delta, p_0, X_0, \emptyset)$ is given by:



Question 4: The PDA $P' = (\{p_0, p_f, q, r\}, \{0,1\}, \{X_0, Z_0, X\}, \delta, p_0, X_0, \{p_f\})$ is given by:



Question 5: The PDA $P = (\{q\}, \{0,1\}, \{0,1,S,A\}, \delta, q, S, \emptyset)$ is given by:



Question 6:

a. Only S is nullable so that we obtain:

$$\begin{aligned} S &\to ASB \mid AB \\ A &\to aAS \mid aA \mid a \\ B &\to SbS \mid bS \mid Sb \mid b \mid A \mid bb \end{aligned}$$

b. There is only one unit production $B \to A$ so that we obtain:

$$
\begin{aligned}
S &\to ASB \mid AB \\
A &\to aAS \mid aA \mid a \\
B &\to SbS \mid bS \mid Sb \mid b \mid aAS \mid aA \mid a \mid bb
\end{aligned}
$$

c. A and B are obviously generating. From $S \to AB$ we conclude that also S is generating. There are no unreachable symbols. As a consequence all symbols are useful.

d. We obtain the following grammar in Chomsky Normal Form:

$$
\begin{aligned}
S &\to AE \mid AB \\
A &\to CF \mid CA \mid a \\
B &\to SG \mid DS \mid SD \mid b \mid CF \mid CA \mid a \mid DD \\
C &\to a \\
D &\to b \\
E &\to SB \\
F &\to AS \\
G &\to DS
\end{aligned}
$$