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# The Search for a $(46, 6, 1)$ Block Design

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**Abstract**

In this paper we report the results of a computer search for a $(46, 6, 1)$ block design. In the incidence matrix of such a design, there must exist a "c4" configuration — 6 rows and 4 columns, in which each pair of columns intersect exactly once, in distinct rows. There can also exist a "c5" configuration - 10 rows and 5 columns, in which each pair of columns intersect exactly once, in distinct rows. Thus the search for $(46, 6, 1)$ designs can be subdivided into two cases, the first of which assumes there is no "c5", and the second of which assumes there is a "c5". After completing the searches for both cases, we found no $(46, 6, 1)$ design.

## 1 Introduction

The existence or non-existence of all designs with $\lambda = 1$ and $k < 6$ are known. In particular, existence results for $k \leq 5$, and for $k = 6$ with $\lambda \neq 1$ are given in [5]. In [1], [3], [4], [5], [7], [8], [9], [10] and [11], the existence of certain designs with $k = 6$ and $\lambda = 1$ are proven. A summary of these results can be found in [2].

In [11], the author also shows that, due to recursive construction, there are only a finite number of designs with $k = 6$ and $\lambda = 1$ which need to be considered before all existence results for designs with these parameters can be proven. The smallest case with the parameters $k = 6$ and $\lambda = 1$ for which it is not known if a design exists is the case $v = 46$. In this paper, we consider this case, by giving details of a computer search for $(46, 6, 1)$ designs.

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There are four sections in the remainder of the paper. In Section 2, we describe the procedure followed in our search for $(46, 6, 1)$ designs, showing how the search may be divided into two cases, the first of which assumes there is no "c5", and the second of which assumes there is a "c5". In Section 3, we describe our method and the results from our search assuming the first case, in which there is no "c5". In Section 4, we describe our method and the results from our search assuming the second case: that is a "c5". In both cases, no designs were found. Since the computer search did not find any $(46, 6, 1)$ design, we conclude that it does not exist.

## 2 Search Procedure

The procedure we shall follow in our search is to attempt to complete the incidence matrix of a $(46, 6, 1)$ design. The incidence matrix of such a design has 46 rows and 69 columns. In each row, there are 9 ones. In each column, there are 6 ones. Each pair of rows intersect in exactly one column.

Any row and column permutations preserve the properties of an incidence matrix. For this reason, we can place the 9 ones of the first row in any columns we wish. Let us place them in columns 1, 2 and 63–69:

$$r1 = 110\,0 \cdots 0\,0000000\,0000000\,1111111.$$

Now consider the second row. The first two rows must intersect in exactly one column. Due to column permutations we can assume that the first two rows intersect in column 1, and that the other ones are placed in columns 3 and 56–62:

$$r2 = 101\,0 \cdots 0\,0000000\,1111111\,0000000.$$

Consider column 2. Each column must contain 6 ones, so due to row permutations we can place a one in row 3. This takes care of the intersection between row 3 and row 1. Due to column permutations we can assume that rows 2 and 3 intersect in column 3, and that the other ones for row 3 are placed in columns 49–55:

$$r3 = 011\,0 \cdots 0\,1111111\,0000000\,0000000.$$

Observe that in the first 3 columns and first $\binom{3}{2} = 3$ rows, each pair of columns intersect exactly once, in distinct rows. We call this type of configuration a "c3" configuration. Next we shall examine if there also must exist a "c4" configuration — 4 columns and $\binom{4}{2} = 6$ rows in which each pair of columns intersect exactly once.

In each of the first three columns, two ones have been placed so far; let us place the remaining ones in each of these columns. In each of the remaining rows, we cannot place more than a single one in the first three columns without creating a situtation in which a pair of rows intersect in more than one column, which is not permitted. Row permutations allow us to place the remaining ones for column 1 in rows 4–7, the remaining ones for column

| | | | | |
|---|---|---|---|---|
| 110 | $0\ldots0$ | 0000000 | 0000000 | 1111111 |
| 101 | $0\ldots0$ | 0000000 | 1111111 | 0000000 |
| 011 | $0\ldots0$ | 1111111 | 0000000 | 0000000 |
| 100 | | | 0000000 | 0000000 |
| 100 | | | 0000000 | 0000000 |
| 100 | | | 0000000 | 0000000 |
| 100 | | | 0000000 | 0000000 |
| 010 | | 0000000 | | 0000000 |
| 010 | | 0000000 | | 0000000 |
| 010 | | 0000000 | | 0000000 |
| 010 | | 0000000 | | 0000000 |
| 001 | | 0000000 | 0000000 | |
| 001 | | 0000000 | 0000000 | |
| 001 | | 0000000 | 0000000 | |
| 001 | | 0000000 | 0000000 | |
| 000 | | | | |
| $\vdots$ | | | | |
| 000 | | | | |

Figure 1: Structure of the Incidence Matrix

2 in rows 8–11, and the remaining ones for column 3 in rows 12–15. Thus we have the partially-filled incidence matrix shown in Fig. 1.

Consider the intersections between each of the partially-filled rows 4–15. We have to ensure that each of rows 4-7 intersects with each of rows 8–15 and that each of rows 8–11 intersects with each of rows 12–15. Thus we need to ensure that 48 row-intersections occur. Clearly these intersections cannot occur in any of the last 21 columns, or, as previously mentioned, any of the first 3 columns. Therefore we are restricted to a total of $69-3-21 = 45$ columns.

Because we require 48 intersections in 45 columns, clearly not all of the row-intersections are disjoint. More specifically, there must be rows $i$, $j$ and $k$, $4 \leq i \leq 7$, $8 \leq j \leq 11$, $12 \leq k \leq 15$, which all intersect in the same column, say column $c$. Observe that column $c$ intersects each of columns 1–3 exactly once. Therefore, rows 1–3 and $i$, $j$ and $k$, together with columns 1–3 and $c$, form a "c4" configuration.

We can therefore assume that the incidence matrix of any $(46, 6, 1)$ design must contain a "c4" configuration. It is also possible for a "c5" configuration to exist, where a "c5" is 5 columns and 10 rows in which each pair of columns intersect exactly once. Therefore we shall divide the search into two cases — the first of which assumes there is no "c5", and the second of which assumes there is a "c5".

Throughout this paper, we label sections of the incidence matrix using two letters. The first letter is either R or C, denoting either row or column. The second letter runs alphabetically starting with A.

# 3 Incidence Matrix with a "c4"

First let us consider the case in which the incidence matrix contains a "c4" configuration but no "c5" configuration. To aid our search, we would like to ensure that the incidence matrix is in a convenient structure. We can assume that the incidence matrix has the structure shown in Fig. 2, as explained below.

We shall place the "c4" configuration in the first six rows and first four columns of the incidence matrix. We shall call these first six rows section RA, and these first four columns section CA. We must ensure that each pair of rows in section RA intersect in exactly one column. There are three pairs of rows which do not already intersect in section CA, therefore we require three columns, each containing exactly two ones in section RA, to create the required intersections. We can assume that these are columns 5–7; we shall call these columns section CB. Each row in section RA requires 6 more ones, and since no other row intersections in section RA are permitted, we require $6*6 = 36$ columns each containing a single one in section RA. We can assume that these are columns 8-43; we shall call these columns section CC. We shall call the remaining $69 - 4 - 3 - 36 = 26$ columns section CD; each of these columns contain only zeroes in section RA.

In section CA, each column contains 3 ones in section RA, thus each of these columns require 3 more ones. In section CA, if we place more than a single one in any row outside of section RA, then that row would intersect a row of section RA in more than one column. Therefore we require $4 * 3 = 12$ rows each containing a single one in section CA. We can assume that these are rows 5–16; we shall call these rows section RB. Section (RB, CB) is filled entirely with zeroes, since if a one were placed in some row, then that row would intersect a row from section RA in two columns. Section (RB, CC) must take care of all remaining intersections between rows of RA and rows of RB. In this section, each one placed in a row of section RB creates an intersection between that row and one row of section RA. Each row of RB intersects exactly three rows of RA in section CA, thus there must be three ones per row in section (RB, CC).

We shall call the remaining $46 - 6 - 12 = 28$ rows section RC; each of these rows contain only zeroes in section CA.

The basic procedure we used to complete the remainder of the incidence matrix is as follows. First, we used BDX to complete the first 18 rows — sections RA and RB — of the incidence matrix.

We subdivided the search based on the number of columns in section (RB,CC) which contain exactly two ones. We call such columns "pairs" since they create an intersection between two rows. Due to intersections of the rows of sections RA and RB, no column in section (RB, CC) can contain more than 2 ones. Let $p$ be the number of columns which contain 2 ones in section (RB, CC). Because there are 12 rows in section RB, each with 3 ones in section CC, $p \leq 12 * 3/2 = 18$.

Let us consider if there are any further restrictions on $p$. Let $n_i$ be the number of columns in section (RB, CD) which contain $i$ ones. If a column containing 4 ones exists in this section, then this column must intersect column 1 in one of rows 7–9, column 2 in one of rows 10–12, column 3 in one of rows 13–15, and column 4 in one of rows 16–18, and thus we have a "c5". Since we are excluding designs containing a "c5" in this part of the search, we need therefore

| | CA<br>4 cols | CB<br>3 cols | CC<br>36 cols | CD<br>26 cols |
|---|---|---|---|---|
| | 1 1 0 0 | 1 0 0 | 1 1 1 1 1 1 0 0 0 0 0 0 . . . 0 0 0 0 0 0 | |
| RA | 1 0 1 0 | 0 1 0 | 0 0 0 0 0 0 1 1 1 1 1 1 . . . 0 0 0 0 0 0 | |
| | 1 0 0 1 | 0 0 1 | | |
| | 0 1 1 0 | 0 0 1 | . . . | 0 ones/col |
| 6 rows | 0 1 0 1 | 0 1 0 | | |
| | 0 0 1 1 | 1 0 0 | 0 0 0 0 0 0 0 0 0 0 0 0 . . . 1 1 1 1 1 1 | |
| | 1 0 0 0 | 0 0 0 | | |
| | 1 0 0 0 | 0 0 0 | | |
| | 1 0 0 0 | 0 0 0 | | |
| RB | 0 1 0 0 | 0 0 0 | 3 ones/row | 5 ones/row |
| | 0 1 0 0 | 0 0 0 | | |
| | 0 1 0 0 | 0 0 0 | | |
| | 0 0 1 0 | 0 0 0 | | |
| 12 rows | 0 0 1 0 | 0 0 0 | * | * |
| | 0 0 1 0 | 0 0 0 | | |
| | 0 0 0 1 | 0 0 0 | | |
| | 0 0 0 1 | 0 0 0 | | |
| | 0 0 0 1 | 0 0 0 | | |
| RC | 0 0 0 0 | | | |
| | . . . | * | * | * |
| 28 rows | 0 0 0 0 | | | |

Figure 2: Structure of Incidence Matrix Containing a “c4”

only consider columns which contain 3 ones or less.

There are 26 columns in section (RB, CD), therefore:

$$n_3 + n_2 + n_1 + n_0 = 26. \tag{1}$$

There are exactly 5 ones in each of the 12 rows of section (RB, CD), therefore there are a total of 60 ones in this section, and:

$$3n_3 + 2n_2 + 1n_1 + 0n_0 = 60. \tag{2}$$

Consider the intersections between each of the rows of section RB. Taking into account the fact that certain rows already intersect in section CA, and that no intersections can occur in section CB, we see that each of rows 7–9 must intersect each of rows 10–18, each of rows 10–12 must intersect each of rows 13–18, and each of rows 13–15 must intersect each of rows 16–18. Therefore a total of 54 intersections are required in sections CC or CD. Any column containing 2 ones takes care of one intersection; any column containing 3 ones takes care of three intersections. Since $p$ is the number of columns of 2 ones in section (RB, CC), each of these takes care of one intersection and therefore:

$$3n_3 + 1n_2 = 54 - p. \tag{3}$$

Subtracting Eq.(2) and 2∗Eq.(1) from Eq.(3), we obtain:

$$1n_1 + 3n_0 = 12 - p.$$

¿From this equation, since the number of any type of columns can never be negative, we have $p \leq 12$. Therefore we have 13 subcases to consider, namely for $0 \leq p \leq 12$.

For each of these 13 subcases, we used BDX in two steps: first to complete section (RB, CC) of the incidence matrix, and then to complete section (RB, CD). Each of these steps produces a number of "images". In fact, since the number of (RB, CC) images for the subcases in which $0 \leq p \leq 5$ is quite small, we treated these as one subcase. Each image was then passed to a specialized program. We ran this specialized program over a network of 22 SPARC-2 and SPARC-10 machines. To do this, we used autoson [6], a tool developed by Brendan D. McKay, for scheduling processes across a network of UNIX workstations.

Note that all that is required for a "c5" to occur is 5 columns and 10 rows in which each pair of columns intersect exactly once. Therefore during this search, in which we assume a "c4" but no "c5", we also test for situations in which a "c5" configuration can occur. If such a situation is found, we can backtrack.

The results of our search are summarized in Table 1. In this table, "Time" refers to the combined time to run BDX and the specialized program. In our search, the specialized program ("c4") was never able to complete more than 29 rows of the incidence matrix, thus no designs were found in this search.

# 4 Incidence Matrix with a "c5"

We now consider the case in which the incidence matrix contains a "c5" configuration. This incidence matrix must have the structure shown in Fig. 3, as explained below.

| #Pairs | #Non-isomorphic (RB, CC) Images | #Non-isomorphic (RB, CD) Images | Average Time / (RB, CC) Image | Total Time |
|---:|---:|---:|---:|---:|
| 0–5 | 206 | 1 148 615 | 1407.8 s | 80.6 hrs |
| 6 | 450 | 3 002 418 | 1000.0 s | 125.0 hrs |
| 7 | 1 237 | 8 628 045 | 889.2 s | 305.6 hrs |
| 8 | 2 987 | 13 598 686 | 502.2 s | 416.7 hrs |
| 9 | 5 958 | 14 517 570 | 302.3 s | 500.0 hrs |
| 10 | 9 403 | 9 148 588 | 127.6 s | 333.3 hrs |
| 11 | 11 425 | 2 516 400 | 36.8 s | 116.7 hrs |
| 12 | 10 416 | 211 843 | 12.5 s | 36.1 hrs |
| Total | 42 078 | | 163.7 s | 1913.9 hrs |

Table 1: Search Results Assuming a "c4"

We shall place the "c5" configuration in the first ten rows and first five columns of the incidence matrix. We shall call these first ten rows section RA, and these first five columns section CA. As before, we must ensure that each pair of rows in section RA intersect in exactly one column. There are fifteen pairs of rows which do not already intersect in section CA. Since the remaining columns in section RA can each have at most 2 ones, we require fifteen columns, each containing exactly 2 ones in section RA, to create the required intersections. We can assume that these are columns 6–20, and shall call these columns section CB. Each row in section RA requires 4 more ones, and since no other row intersections in section RA are permitted, we require $10 * 4 = 40$ columns each containing a single one in section RA. We can assume that these are columns 21–60, and shall call these columns section CC. We shall call the remaining $69 - 5 - 15 - 40 = 9$ columns section CD; each of these columns contain only zeroes in section RA.

In section CA, each column contains 4 ones in section RA, thus each of these columns require 2 more ones. In section CA, if we place more than a single one in any row outside of section RA, then that row would intersect a row of section RA in more than one column. Therefore we require $5 * 2 = 10$ rows each containing a single one in section CA. We can assume that these are rows 11–20; we shall call these rows section RB.

For each row in section RB, let $b$, $c$ and $d$ be the number of ones in sections CB, CC and CD respectively. Since each row has 9 ones and section CA contains a single one, we have:

$$1 + b + c + d = 9.$$

By counting intersections with the 10 rows in section RA, we have

$$4 + 2b + c + 0d = 10,$$

which simplifies to

$$2b + c = 6. \tag{4}$$

| | CA<br>5 cols | CB<br>15 cols | CC<br>40 cols | CD<br>9 cols |
|---|---|---|---|---|
| RA<br>10 rows | 1 1 0 0 0<br>1 0 1 0 0<br>1 0 0 1 0<br>1 0 0 0 1<br>0 1 1 0 0<br>0 1 0 1 0<br>0 1 0 0 1<br>0 0 1 1 0<br>0 0 1 0 1<br>0 0 0 1 1 | 2 ones/col | 1 1 1 1 0 0 0 0 . . . 0 0 0 0<br>0 0 0 0 1 1 1 1 . . . 0 0 0 0<br>. . .<br>0 0 0 0 0 0 0 0 . . . 1 1 1 1 | 0 ones/col |
| RB<br>10 rows | 1 0 0 0 0<br>1 0 0 0 0<br>0 1 0 0 0<br>0 1 0 0 0<br>0 0 1 0 0<br>0 0 1 0 0<br>0 0 0 1 0<br>0 0 0 1 0<br>0 0 0 0 1<br>0 0 0 0 1 | 0,1,2 or 3<br>ones/row<br>* | 6,4,2 or 0<br>ones/row<br>* | 2,3,4 or 5<br>ones/row<br>* |
| RC<br>26 rows | 0 0 0 0 0<br>. . .<br>0 0 0 0 0 | * | * | * |

Figure 3: Structure of Incidence Matrix Containing a "c5"

Since $c \geq 0$, Eq.(4) implies $0 \leq b \leq 3$. Solving the equations, we have the following possibilities: $\{b = 0, c = 6, d = 2\}$, $\{b = 1, c = 4, d = 3\}$, $\{b = 2, c = 2, d = 4\}$ and $\{b = 3, c = 0, d = 5\}$

We shall call the remaining $46 - 10 - 10 = 26$ rows section RC; each of these rows contain only zeroes in section CA.

Consider section (RB, CD) further. In this section there are at least 2 ones per row; since there are 10 rows, there must be at least 20 ones in total in this section. Since there are 9 columns, at least 1 column must contain 3 ones or more. Furthermore, there are at most 5 ones per column since 5 (disjoint) pairs of these rows already intersect in section CA.

We divide the search assuming a "c5" configuration into three subcases, based on the maximum number of ones in any column of section (RB, CD):

1. At least one column in (RB, CD) contains 5 ones, and all other columns contain at most 5 ones,

2. At least one column in (RB, CD) contains 4 ones, and all other columns contain at most 4 ones, and

| Case | #Subcases | Total Time |
|---|---|---|
| 1 | 147 | < 1 hrs |
| 2 | 449 | 8 hrs |
| 3 | 1111 | 22 hrs |
| 4 | 1500 | 57 hrs |
| 5 | 354 | 37 hrs |
| 6 | 144 | 6 hrs |
| 7 | 309 | 163 hrs |
| 8 | 1753 | 282 hrs |
| 9 | 1493 | 78 hrs |
| Total | 7260 | 653 hrs |

Table 2: Results Assuming a Column of 5 Ones

3. Each column contains at most 3 ones.

The basic procedure to complete the search is as follows. For each of the 3 subcases, we fill in selected portions of the incidence matrix using BDX. This step creates a number of "images", each of which is then passed on to a specialized program, which attempts to complete the remainder of the incidence matrix.

### 4.1 A column of 5 ones

We can assume that column 61 in section (RB, CD) contains exactly 5 ones and that all other columns in this section contain at most 5 ones. Up to isomorphism, there is only one way to complete column 61, namely with ones in rows 11, 13, 15, 17, 19 and 21. Completing the remainder of rows 11, 13, 15, 17 and 19 gives 9 non-isomorphic cases.

For each of these cases, we further complete selected portions of the incidence matrix using BDX. These portions are selected based on the orbit structure of the automorphism group of the partially completed matrix, with the goal of ensuring an efficient isomorph rejection. For example, the automorphism group of the first of these 9 non-isomorphic cases has size 48. There is a row orbit containing only rows 12 and 21. Completing these two rows generates 147 non-isomorphic subcases, 113 of which have a trivial automorphism group. Since a large majority of these subcases is rigid, isomorph rejection is no longer effective and we switch over to a specilized program. This step of "using" up the automorphism group to generate another level of subcases also makes it easier to run them on a network of processors as independent jobs.

The results of the search are summarized in Table 2. In this table, the time is normalized to an execution on a SPARC-10 computer.

### 4.2 A column of 4 ones

We can assume that column 61 in section (RB, CD) contains exactly 4 ones and that all other columns in this section contain at most 4 ones. Up to isomorphism, there is only one way to complete column 61, namely with ones in rows 11, 13, 15, 17, 21 and 22. Completing the remainder of rows 11, 13, 15 and 17 gives 33 non-isomorphic cases.

The results of the search by assuming at least one column of 4 ones, but no column of 5 ones, are summarized in Table 3. For efficiency consideration, we have to handle cases with more than one column of 4 ones separately. In the table, cases 1–33 are the original 33 cases, assuming exactly 1 column containing 4 ones. Cases $i1_j$–$i4_j$ are subcases assuming exactly $j$ columns containing 4 ones. Note that the overall number of subcases considered is 138 955 and the overall execution time was 89 986 hours. Again, the time is normalized to an execution on a SPARC-10 computer.

### 4.3 Columns of 3 ones

Next we consider the case in which each column in section (RB, CD) of the incidence matrix contains at most 3 ones. We subdivide this case based on the number of columns in this section containing exactly 3 ones, taking into account the restriction mentioned earlier that the total number of ones in this section is at least 20. Therefore we have eight subcases, since there must be at least two, and at most nine, columns containing exactly 3 ones in section (RB, CD).

For each of these subcases, we used BDX to complete all columns in section (RB, CD) which contain exactly 3 ones. For the subcase which has two columns of 3 ones, we also filled in other columns in this section. Each non-isomorphic completion is then handled by a specialized program.

The results of a search assuming columns of 3 ones, but no column with 4 or 5 ones, are summarized in Table 4. The time shown in this table include both the time required by BDX, and the time required by the specialized program.

## 5 Summary

We have shown how the search for a $(46, 6, 1)$ design may be divided into two cases, the first of which assumes a "c4" but no "c5", and the second of which assumes a "c5". We have completed computer searches of both cases and found no designs. Therefore, we conclude that there is no $(46, 6, 1)$ design.

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| Case | #Subcases | Total Time |
|---:|---:|---:|
| 1 | 298 | 13 hrs |
| 2 | 150 | 8 hrs |
| 3 | 214 | 66 hrs |
| 4 | 541 | 5 hrs |
| 5 | 306 | 39 hrs |
| 6 | 99 | 33 hrs |
| 7 | 404 | 55 hrs |
| 8 | 48 | 39 hrs |
| 9 | 539 | 677 hrs |
| 10 | 418 | 77 hrs |
| 11 | 405 | 86 hrs |
| 12 | 106 | 21 hrs |
| 13 | 723 | 603 hrs |
| 14 | 901 | 9 hrs |
| 15 | 44 | 24 hrs |
| 16 | 535 | 695 hrs |
| 17 | 44 | 22 hrs |
| 18 | 918 | 13 hrs |
| 19 | 393 | 65 hrs |
| 20 | 1734 | 1199 hrs |
| 21 | 3211 | 10 hrs |
| 22 | 1575 | 821 hrs |
| 23 | 2037 | 3910 hrs |
| 24 | 1803 | 6 hrs |
| 25 | 535 | 321 hrs |
| 26 | 3317 | 18 hrs |
| 27 | 550 | 901 hrs |
| 28 | 5927 | 3507 hrs |
| 29 | 2422 | 284 hrs |
| 30 | 2268 | 5747 hrs |
| 31 | 4215 | 1002 hrs |
| 32 | 2077 | 11237 hrs |
| 33 | 6970 | 3293 hrs |
| Total | 45727 | 34806 hrs |

| Case | #Subcases | Total Time |
|---:|---:|---:|
| $i1_2$ | 3093 | 5417 hrs |
| $i2_2$ | 4804 | 33417 hrs |
| $i3_2$ | 3267 | 1570 hrs |
| $i4_2$ | 25204 | 10781 hrs |
| $i1_3$ | 12579 | 970 hrs |
| $i2_3$ | 408 | 1523 hrs |
| $i3_3$ | 408 | 1388 hrs |
| $i1_4$ | 1787 | 7 hrs |
| $i2_4$ | 1748 | 26 hrs |
| $i3_4$ | 38331 | 79 hrs |
| $i1_5$ | 1599 | 2 hrs |
| Total | 93228 | 55180 hrs |

Table 3: Results Assuming a Column of 4 Ones

| #Cols of 3 Ones | #Subcases | Total Time |
|---:|---:|---:|
| 9 | 1499 | 400 hrs |
| 8 | 3735 | 4159 hrs |
| 7 | 3730 | 13937 hrs |
| 6 | 2061 | 26107 hrs |
| 5 | 659 | 24609 hrs |
| 4 | 152 | 9672 hrs |
| 3 | 25 | 1390 hrs |
| 2 | 902 | 68 hrs |
| Total | 12763 | 80342 hrs |

Table 4: Results Assuming a Column of 3 Ones


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