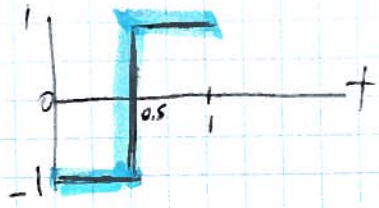
(4) (COSC 4P98, B. Ross)

EXAMPLE 1: SQUARE WAVE



: calculate a_n, b_n

note: $\int (\sin nx)\,dx = -\frac{\cos nx}{n}$

$\int (\cos nx)\,dx = \frac{\sin nx}{n}$

$$a_n = 2\int_0^1 f(t)\cos 2\pi n t\,dt$$

$$= 2\left[\int_0^{0.5} f(t)\cos 2\pi n t\,dt + \int_{0.5}^{1} f(t)\cos 2\pi n t\,dt\right]$$

$$= 2\left[\int_0^{0.5} (-1)\cos 2\pi n t\,dt + \int_{0.5}^{1} (+1)\cos 2\pi n t\,dt\right]$$

$$= 2\left(\left[\frac{-\sin 2\pi n t}{2\pi n}\right]_0^{\frac{1}{2}} + \left[\frac{\sin 2\pi n t}{2\pi n}\right]_{\frac{1}{2}}^{1}\right)$$

$$= 2\left(\left[\frac{-\sin \pi n}{2\pi n} + \frac{\sin 0}{2\pi n}\right] + \left[\frac{\sin 2\pi n}{2\pi n} - \frac{\sin \pi n}{2\pi n}\right]\right)$$

($t=\frac{1}{2}$) ($t=0$) ($t=1$) ($t=\frac{1}{2}$)

$= 2[0+0+0+0] = 0$: no coefficients (ie. no phase shifts)

$$b_n = 2\int_0^1 f(t)\sin 2\pi n t\,dt$$

$$= 2\left[\int_0^{0.5} f(t)\sin 2\pi n t\,dt + \int_{0.5}^{1} f(t)\sin 2\pi n t\,dt\right]$$

(over)

(Cosc 4998, B. Ross)

⑤ Fourier (SQUARE)

$$2\left[\int_0^{.5}(-1)\sin 2\pi n t\,dt + \int_{.5}^{1}(+1)\sin 2\pi n t\,dt\right]$$

$$= 2\left(\left[\frac{\cos 2\pi n t}{2\pi n}\right]_0^{\frac{1}{2}} + \left[\frac{-\cos 2\pi n t}{2\pi n}\right]_{\frac{1}{2}}^{1}\right)$$

$$= 2\left(\left[\frac{\cos\pi n}{2\pi n} - \frac{\cos 0}{2\pi n}\right] + \left[\frac{\cos 2\pi n}{2\pi n} - \frac{-\cos\pi n}{2\pi n}\right]\right)$$

$$= \cancel{2}\left(\frac{2\cos\pi n}{\cancel{2}\pi n} - \frac{1}{\cancel{2}\pi n} - \frac{1}{\cancel{2}\pi n}\right) = \frac{2}{\pi n}(\cos\pi n - 1)$$

n even ⓔ

$\frac{2}{\pi n}(\cos\pi n - 1)$

$= \frac{2}{\pi n}(1-1) = 0$

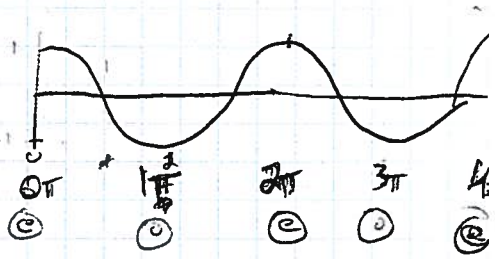
n odd ⓞ

$\frac{2}{\pi n}(\cos\pi n - 1)$

$= 2/\pi n\,(-1-1)$

$= -4/\pi n$

$\Rightarrow$



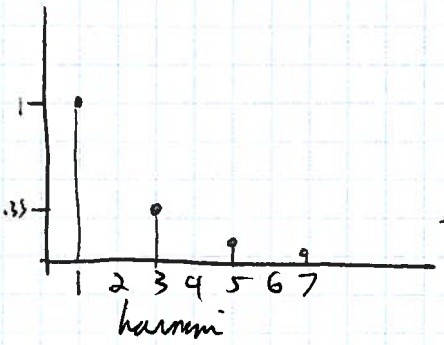
so: $f(t) = \sum a_n \cos 2\pi n t + b_n \sin 2\pi n t$

a_n: 0 (even)

b_n: 0 (even), $-\frac{4}{\pi n}$ (odd)

$$f(t) = \frac{-4}{1\pi}\sin 2\pi 1t - \frac{4}{3\pi}\sin 2\pi 3t - \frac{4}{5\pi}\sin 2\pi 5t \cdots$$

↑ fundamental ↑ 3rd harm ↑ 5th harm



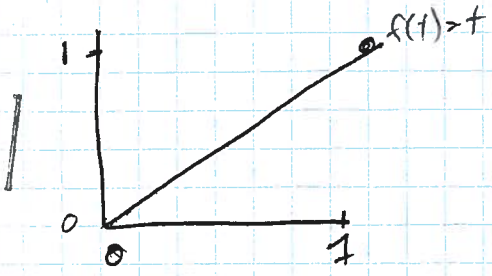
etc.

scale $\frac{1}{n}$

($-\frac{4}{\pi}$ is constant scale) (can be removed)

(6) Fourier series 05 (cosc 4P98, B. Ross)

eg 2 - Sawtooth wave



find a_n, b_n

$a_n = \dots = 0$

$b_n = 2\int_0^1 f(t)\sin 2\pi n t\,dt$

$= 2\int_0^1 t\sin 2\pi n t\,dt$

$= 2\left[\frac{\sin 2\pi n t}{(2\pi n)^2} - \frac{t\cos 2\pi n t}{2\pi n}\right]_0^1$

from "table"

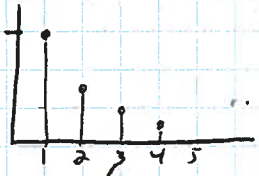
$\int t\sin kt\,dt = \frac{\sin kt}{k^2} - \frac{t\cos kt}{k}$

$= 2\left(\left[\frac{\sin 2\pi n}{(2\pi n)^2} - \frac{\cos 2\pi n}{2\pi n}\right] - \left[\frac{\sin 0}{(2\pi n)^2} + \frac{0\cos 0}{2\pi n}\right]\right)$

$= 2\left(0 - \frac{1}{2\pi n} - 0 - 0\right) = -\frac{1}{n\pi}$

so $f(t) = \sum_{n=0}^{\infty} a_n\cos 2\pi n t + b_n \sin 2\pi n t$

$= \frac{1}{1}\sin 2\pi 1t + \frac{1}{2}\sin 2\pi 2t + \frac{1}{3}\sin 2\pi 3t + \dots$ ($\sin$ use + a - freq)



scale $\rightarrow \left(\frac{1}{\pi}\right)$