Revised: Feb 22, 2012

# Fourier Series and Discrete Fourier Transformation Side-by side

Fourier Series:

$$f(t) = \alpha_0 + \sum_{n=1}^{\infty}\left( \mathrm{a}_{\mathrm{n}} \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right)$$

$$a_n = \frac{2}{T}\int_0^T \mathrm{f(t)}\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$b_n = \frac{2}{T}\int_0^T \mathrm{f(t)}\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$a_0 = \frac{1}{T}\int_0^T \mathrm{f(t)}\, dt$$

Discrete Fourier Transformation (DFT):

$$f(t) = \alpha_0 + \sum_{n=1}^{T}\left( \mathrm{a}_{\mathrm{n}} \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right)$$

$$a_n = \frac{2}{T}\sum_{k=0}^{T-1} x[k]\cos\left(\frac{2\pi nk}{T}\right)$$

$$b_n = \frac{2}{T}\sum_{k=0}^{T-1} x[k]\sin\left(\frac{2\pi nk}{T}\right)$$

$$a_0 = \frac{1}{T}\sum_{k=0}^{T-1} x[k]$$

Where:

- **t** = time (from 0,…,T-1)
- **T** = total # samples
- **f(t)** is amplitude of wave at time t
- **n** = harmonic number (max possible harmonic is T/2, or Nyquist frequency)
- **x[k]** is sample at index k in wave table

a) To compute amplitude and phase of the nth harmonic...

$$amplitude_n = \sqrt{a_n^{\,2} + b_n^{\,2}} \qquad\qquad phase_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

b) To reconstruct the wave from the harmonics...

You simply take the first equation above, and recompute the wave at each moment of time: t = 0, 1, 2, ..., T-1. You compute what each harmonic value is at that time, and sum the results. The overall sum is the sample value at time t. Remember to add the a0 value to all values!

<u>Reference</u>:

Class notes.
Chapter 3 in *Who is Fourier?*
Pay special attention to the exercises on pages 131-134!