

COSC 5P02 - Logic in Computer Science

Term Test 3

Question 1 (10 marks): Suppose p and q are unary predicate symbols. Find a derivation for $\exists x:(p(x) \wedge q(x)) \rightarrow \exists x:p(x) \wedge \exists x:q(x)$ in natural deduction. Justify the application of a rule whenever necessary (variable conditions).

Solution:

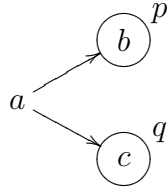
$$\frac{\frac{\frac{[p(x) \wedge q(x)]^2}{p(x)} \wedge E1}{\exists x:p(x)} \exists I \quad \frac{\frac{[p(x) \wedge q(x)]^2}{q(x)} \wedge E2}{\exists x:q(x)} \exists I}{\exists x:p(x) \wedge \exists x:q(x)} \wedge I}{\frac{[\exists x:(p(x) \wedge q(x))]^1}{\exists x:p(x) \wedge \exists x:q(x)} \exists E^{2,a}}{\exists x:(p(x) \wedge q(x)) \rightarrow \exists x:p(x) \wedge \exists x:q(x)} \rightarrow I^1$$

a) The application of the $\exists E$ -rule is valid because the formulas $\exists x:p(x) \wedge \exists x:q(x)$ does not contain x free. There is no assumption in the subtree besides $p(x) \wedge q(x)$.

Question 2 (10 marks): Show that the formula $\diamond p \wedge \diamond q \rightarrow \diamond(p \wedge q)$ is not true.

Hint: A model with 3 elements is sufficient.

Solution: Consider the following model \mathcal{M} :



Then $\mathcal{M}, a \models \Diamond p \wedge \Diamond q$ because of Rab and $b \in v(p)$ resp. Rac and $c \in v(q)$. But we have $\mathcal{M}, a \not\models \Diamond(p \wedge q)$ because no world is related to a in which $p \wedge q$ is true.