

COSC 5P02 - Logic in Computer Science

Term Test 3

Question 1: Consider the following two modular formulas:

1. $p \rightarrow \diamond p$.
2. $p \rightarrow \square \diamond p$.

Provide models $\mathcal{M}_0 - \mathcal{M}_2$ with the following properties (5 marks each):

- (a) In \mathcal{M}_0 none of the formulas is true.
- (b) In \mathcal{M}_1 the Formula 1. is true and Formula 2. is false.
- (c) In \mathcal{M}_2 the Formula 2. is true and Formula 1. is false.
- (d) In \mathcal{M}_4 both formulas are true.

Solution: In each of the examples we choose $W = \{a, b\}$ and $v(p) = \{a\}$. Since both formulas are of the form $p \rightarrow \dots$ it is sufficient to check whether the formula is true at state a (both formulas are always true at b since b is not in p).

- (a) In the model $a \longrightarrow b$ both formulas are not true for a since neither does a have a successor in p nor does the successor b of a has a successor in p .

- (b) In the model $\begin{array}{c} \circlearrowleft \\ a \longrightarrow b \end{array}$ formula 1. is true at a since a is a successor of a satisfying p . Formula 2. is false at a since the successor b of a does not have a successor in p .
- (c) Consider the model $\begin{array}{c} a \rightleftarrows b \end{array}$. As in the first model Formula 1. is false at state a . The second formula is true at a since the only successor b of a has a successor (a) in p .
- (d) Last but not least, in the model $\begin{array}{c} \circlearrowleft \\ a \rightleftarrows b \end{array}$ both formulas are true at a . a is a successor of a in p , and both successors of a (a and b) have a successor a in p .