

COSC 5P02 - Logic in Computer Science

Term Test 1

Question 1: Find a derivation of $((\neg p \vee q) \wedge p) \rightarrow q$ in natural deduction (10 marks).

Solution:

$$\frac{\frac{\frac{[(\neg p \vee q) \wedge p]^1}{\neg p \vee q} \wedge E1 \quad \frac{[\neg p]^2 \quad \frac{[(\neg p \vee q) \wedge p]^1}{p} \wedge E2}{\perp} \neg E}{\frac{\perp}{q} \text{PBC}} \vee E2}{\frac{q}{((\neg p \vee q) \wedge p) \rightarrow q} \rightarrow I^1} \vee E^2$$

Question 2: Find a formula in disjunctive normal form equivalent to the formula $(p \rightarrow \neg q) \wedge \neg(q \vee \neg p)$ (7 marks). Is the formula satisfiable (3 marks)?

Solution:

$$\begin{aligned} & (p \rightarrow \neg q) \wedge \neg(q \vee \neg p) \\ \rightsquigarrow & (\neg p \vee \neg q) \wedge \neg(q \vee \neg p) \\ \rightsquigarrow & (\neg p \vee \neg q) \wedge \neg q \wedge p \\ \rightsquigarrow & (\neg p \wedge \neg q \wedge p) \vee (\neg q \wedge \neg q \wedge p) \end{aligned}$$

Yes, the formula is satisfiable because the second conjunction does not have a pair $x, \neg x$.