

COSC 5P02 - Logic in Computer Science

Term Test 1

Question 1:

- Show that $p \vee q \models \neg p \rightarrow q$ using a truth table (5 marks).
- Give a derivation $p \vee q \vdash \neg p \rightarrow q$ in natural deduction (5 marks).

Solution:

a.

p	q	$p \vee q$	$\neg p$	$\rightarrow q$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	F

The formula $p \vee q$ is true in Row 1-3. In each case $\neg p \rightarrow q$ is also true so that $p \vee q \models \neg p \rightarrow q$ follows.

b.

$$\begin{array}{c}
 \frac{p \vee q \quad [p]^2}{p} \vee E^2 \\
 \frac{[q]^2 \quad [\neg q]^1}{\perp} \neg E \\
 \frac{\perp}{\neg p \rightarrow q} \rightarrow I^1
 \end{array}$$

Question 2: Find a conjunctive normal form equivalent to the formula $(p \vee q) \rightarrow (\neg p \rightarrow q)$ (7 marks). Is this formula a tautology (3 marks)?

Solution:

$$\begin{aligned} & (p \vee q) \rightarrow (\neg p \rightarrow q) \\ \rightsquigarrow & \neg(p \vee q) \vee \neg\neg p \vee q \\ \rightsquigarrow & (\neg p \wedge \neg q) \vee \neg\neg p \vee q \\ \rightsquigarrow & (\neg p \wedge \neg q) \vee p \vee q \\ \rightsquigarrow & (\neg p \vee p \vee q) \wedge (\neg q \vee p \vee q) \end{aligned}$$

This formula is a tautology because each disjunction contains a pair $x, \neg x$ for some propositional variable x .