

COSC 5P02 - Logic in Computer Science

Term Test 1

Question 1:

1. Show that $p \vee q, \neg p \models q$ using a truth table (4 marks).
2. Give a derivation of $p \vee q, \neg p \vdash q$ in natural deduction (8 marks).

Solution:

1.

p	q	$p \vee q$	$\neg p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

Just in line 3 both assumptions are true - here the conclusion is also true.

2.

$$\frac{\frac{p \vee q}{q} \quad \frac{\frac{\neg p \quad [p]^1}{\perp} \neg E \quad \frac{\perp}{q} \text{PBC}}{[q]^1} \vee E^1}{q}}$$

Question 2: Find a conjunctive normal form equivalent to the formula $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$ (6 marks). Is this formula a tautology (2 marks)?

Solution:

$$\begin{aligned} & ((p \rightarrow q) \wedge \neg q) \rightarrow \neg p \\ \rightsquigarrow & \neg((\neg p \vee q) \wedge \neg q) \vee \neg p \\ \rightsquigarrow & (p \wedge \neg q) \vee q \vee \neg p \\ \rightsquigarrow & (p \vee q \vee \neg p) \wedge (\neg q \vee q \vee \neg p) \end{aligned}$$

Yes, the formula is a tautology since both disjunctions include a pair $x, \neg x$.