

Discrete Fourier Transformation (DFT):

a) To convert samples $x[k]$ to harmonics...

$$f(t) = \sum_{n=1}^{T/2} \left(a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right) \right) + a_0$$

$$a_n = \left(\sum_{k=0}^{T-1} x[k] \cos\left(\frac{2\pi nk}{T}\right) \right) \cdot \left(\frac{2}{T} \right)$$

$$b_n = \left(\sum_{k=0}^{T-1} x[k] \sin\left(\frac{2\pi nk}{T}\right) \right) \cdot \left(\frac{2}{T} \right)$$

Where:

t = time (from 0, ..., T-1)

T = total # samples

$f(t)$ is amplitude of wave at time t

n = harmonic number (max possible harmonic is $T/2$, or Nyquist frequency)

$x[k]$ is sample at index k in wave table

b) To reconstruct wave from the harmonics...

You simply take the first equation above, and recompute the wave at each moment of time: $t = 0, 1, 2, \dots, T-1$. You compute what each harmonic value is at that time, and sum the results. The overall sum is the sample value at time t . Remember to add the a_0 value to all values!

(over)

c) To compute amplitude and phase of the nth harmonic...

$$\text{amplitude}_n = \sqrt{a_n^2 + b_n^2}$$

$$\text{phase}_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

Reference:

Chapter 3 in *Who is Fourier?*

Pay special attention to the exercises on pages 131-134!