Computational Geometry: Linear Equation Coefficients

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Given: Two unique points P1 (x1, y1) and P2 (x2, y2) on a line.

What is the equation for the line?

The following can be derived from a base equation:

$$(y-y1) / (x-x1) = (y2-y1) / (x2-x1)$$

Using the definition of m (slope):

$$m = (y2 - y1) / (x2 - x1)$$

The base equation is rewritten:

$$(y-y1) / (x-x1) = m$$

(a) Form: y = mx + b

By solving base equation for y, you get:

$$y = m(x-x1) + y1$$
or $$y = mx + y1 - mx1$$

Then:

$$m = (y2 - y1) / (x2 - x1)$$

$$b = y1 - mx1$$
$$= y1 - (x1 (y2 - y1) / (x2 - x1)).$$

Note: beware of vertical lines, because m cannot be computed (divide by zero).

(b) Form: Ax + By + C = 0

By substituting m and b in base equation with coefficients from (a), after simplifying you get:

A = (y1 - y2)
B = (x2 - x1)
C = x1y2 - x2y1

Note that this form does not have to double check if line is vertical.
Note: If you do your own algebraic manipulation, you may find your terms are 'negative' to the above coefficients for A, B and C. This can happen, and it is still correct. It's akin to multiplying both sides of the equation by -1.

References:

Linear Equation: https://en.wikipedia.org/wiki/Linear_equation

Equation of plane: https://keisan.casio.com/exec/system/1223596129